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ABSTRACT

This issue contains abstracts and critical comments for ten mathematics education journal articles, plus an editorial on teacher education needs by Thomas J. Cooney. Two articles focus on problem solving; the remainder concern instruction about and with computers, geometric perceptions, preservice teachers' conceptions of volume, attitudes toward mathematics, time on task, mastery learning and student teams, teachers' conceptions and practices, and small-group interaction. References to mathematics education research reported in "Resources in Education" (RIE) and "Current Index to Journals in Education" (CIJE) from October through December 1984 are also included. (MNS)

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INVESTIGATIONS IN MATHEMATICS EDUCATION
Volume 18, Number 2 - Spring 1985

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INVESTIGATIONS IN MATHEMATICS EDUCATION

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An editorial comment. . .

Reflecting. . .

Thomas J. Cooney
University of Georgia

The current rhetoric in mathematics education raises serious questions about the quality of mathematics teaching in the schools today. Although the situation may not be as desperate as some of the rhetoric suggests, I believe the claims have substance. How can we as a profession address the need to improve the quality of mathematics instruction and what does this issue have to do with investigations in mathematics education?

Bush (1982) found that the preservice teachers he studied used only a smattering of the knowledge gained from their methods course during student teaching. Further, what was used seemed to have its roots in experiences prior to the methods course. Thompson (1984) found that teachers' conceptions of mathematics and the teaching of mathematics influence, albeit in subtle ways, their instructional practices. Hence, if preservice teachers tend to embrace only those aspects of teaching that they have experienced as students, an influence that Van Fleet (1979) calls enculturation, and if their conceptions of mathematics and teaching are reflections of what are likely to be conservative styles of teaching (where inquiry is anything but central), then teacher education faces a significant impediment to improving the teaching of mathematics. Within the context of this concern, several issues arise related to investigations in mathematics education.

One of the issues has to do with how we go about the business of understanding what interns think is important about the teaching of mathematics and what they believe is the nature of the subject they

teach. In short, we should try to understand as much about "where they are coming from" as we urge them to consider where their "students are coming from". Efforts to understand meanings, values and conceptions bring into question a contrast between what Mitroff and Kilman (1978) describe as analytic and humanistic methodologies, the former being a characteristic of research in the "hard sciences" and the latter being a characteristic of anthropological research. Analytic methodologies provide us with mechanisms to consider comparisons and to establish generalizations that attempt to describe more than singular situations. Such generalizations can be useful in helping us recognize patterns of teaching behavior and by providing us with clues to possible alternatives when things go awry in the classroom.

Humanistic methodologies yield different sorts of generalizations, ones that Stake (1978) calls "naturalistic generalizations." He argues that generalizing from a particular is a "natural" way to generalize. Such generalizations are derived from tacit knowledge, knowledge that is a composite of shared meanings among humankind. As Eisner (1981) put it, there is generality in the particular. Blake said it so beautifully in the following way:

> To hold the world in a grain of sand
> And heaven in a wild flower;
> To hold infinity in the palm of your hand,
> And Eternity in an hour.

One of the most celebrated and interesting examples of the power of naturalistic generalizations resulted from Erlwanger's study (1973) of Benny's misconceptions and the means Benny used to solve problems. Is it not the case that the study of Benny represented more than a single particular and conveyed a more general message about approaches to mathematics education in which errorless expression is valued?

It seems to me that our efforts in teacher education must somehow account for and take into consideration the meanings held by the students we teach. In order to do this, I feel it is necessary for the researcher to minimize the distance between himself/herself and the interns and to use a methodology that maximizes potential for understanding meanings and beliefs. The preservation of perceived objectivity through the use of analytic methodologies pales in the face of the possibility of capturing those meanings and beliefs. This suggests, I contend, that we engage in reflective activity about possible alternative paradigms for generating questions and for seeking appropriate methods for answering those questions.

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Spring 1985

An editorial comment.... THOMAS J. COONEY iii

Battista, Michael T. and Steele, Kathleen J.
THE EFFECT OF COMPUTER-ASSISTED AND COMPUTER PROGRAMMING INSTRUCTION ON THE COMPUTER LITERACY OF HIGH ABILITY FIFTH GRADE STUDENTS. School Science and Mathematics 84: 649-658; December 1984.
Abstracted by H. LAVERNE THOMAS 1

Cooper, Lynn A. and Shepard, Roger N.
TURNING SOMETHING OVER IN THE MIND. Scientific American 251: 106-107, 110-114; December 1984.
Abstracted by DAIYO SAWADA 5

Darch, Craig; Carnine, Doug; and Gersten, Russell.
EXPLICIT INSTRUCTION IN MATHEMATICS PROBLEM SOLVING. Journal of Educational Research 77: 351-359; July/August 1984.
Abstracted by ERIC W. HART and HAROLD L. SCHOEN 14

Enochs, L. G. and Gabel, D. L.
PRESERVICE ELEMENTARY TEACHERS' CONCEPTIONS OF VOLUME. School Science and Mathematics 84: 670-680; December 1984.
Abstracted by MICHAEL T. BATTISTA 20

Minato, Saburoh and Yanase, Shyoichi. ON THE RELATIONSHIP BETWEEN STUDENTS' ATTITUDES TOWARDS SCHOOL MATHEMATICS AND THEIR LEVELS OF INTELLIGENCE. Educational Studies in Mathematics 15: 313-320; August 1984.
Abstracted by CHARLEEN M. DERIDDER 28

Muth, K. Denise. SOLVING ARITHMETIC WORD PROBLEMS: ROLE OF READING AND COMPUTATIONAL SKILLS. Journal of Educational Psychology 76: 205-210; April 1984.
Abstracted by DALE DROST 33

Seifert, Edward H. and Beck, John J., Jr.
RELATIONSHIPS BETWEEN TASK TIME AND LEARNING GAINS IN SECONDARY SCHOOLS. Journal of Educational Research 78: 5-10; September/October 1984.
Abstracted by MORRIS LAI 38

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Abstracted by GEORGE W. BRIGHT 42

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Abstracted by THOMAS GIBNEY 48

Webb, Noreen M. STABILITY OF SMALL GROUP INTERACTION AND ACHIEVEMENT OVER TIME. Journal of Educational Psychology 76: 211-224; April 1984.
Abstracted by KENNETH E. VOS 56

Mathematics Education Research Studies Reported in Journals as Indexed by Current Index to Journals in Education (October-December 1984) 61

Mathematics Education Research Studies Reported in Resources in Education (October-December 1984). 64

Battista, Michael T.; Steele, Kathleen J. THE EFFECT OF COMPUTER-ASSISTED AND COMPUTER PROGRAMMING INSTRUCTION ON THE COMPUTER LITERACY OF HIGH ABILITY FIFTH GRADE STUDENTS. School Science and Mathematics 84: 649-658; December 1984.

Abstract and comments prepared for I.M.E. by H. LAVERNE THOMAS, State University of New York - College at Oneonta.

1. Purpose

The purpose of the study was "to investigate the effects of two commonly used types of computer based instruction, CAI and programming instruction, on the feelings and knowledge students have about computers" (p. 650). Specifically, the study sought to determine (1) the differential effect of CAI instruction in mathematics and computer programming instruction on computer literacy in both the affective and cognitive domains and (2) the extent to which students receiving computer programming instruction become knowledgeable about computer capabilities and uses of computers without such direct instruction.

2. Rationale

This study is rooted in the contemporary recommendations for the teaching of computer literacy as well as in the controversy as to what properly constitutes computer literacy. In particular, the disagreement as to the relationship of "hands-on" programming experience with computers to the achievement of computer literacy goals is noted. Such disagreement, coupled with the perceived lack of knowledge about the effect of hand-on programming experience vis-a-vis computer literacy, is the stated basis for the study.

3. Research Design and Procedures

A control group and two treatment groups (CAI and Programming) were employed. The Control Group and the CAI treatment group subjects (24

in each) were selected on the basis of their (high) IQ scores from subjects in a previous study by Steele (1981). The 24 subjects in the Programming group were all of the subjects, selected on the basis of high academic achievement, in a prior study by Battista (1981). The groups were deemed comparable as high-ability subjects, based upon IQ test scores.

Comparable subject matter in mathematics was studied by the Control and CAI groups, using individualized paper-and-pencil materials and CAI materials, respectively, over the course of a school year (10 minutes, twice a week). The Programming group explored BASIC programming concepts (one 37-minute period per week) over most of a school year (8 weeks were missed after midyear). This work was largely individualized, with students working in pairs at a computer. Instructor effect was deemed minimal because of the individualized nature of the experiences of all groups.

Post-testing used the Minnesota Computer Literacy Awareness Assessment (MCLAA) instrument, with the Programming group tested on a subtest of 56 items and the other groups tested on the full test of 83 items. The common subtest was used as a measure of computer literacy outcomes for the study, with subscales for affective (20 five-point Likert-scale items) and cognitive outcomes (36 items). Chronbach Alpha reliability coefficients are quoted for the main test and each subscale, with a low reliability (.67) noted for the cognitive subscale.

4. Findings

Comparison of means for the three groups showed the Programming group had a higher mean on the affective subscale, with the sharpest difference appearing on the Enjoyment component of that subscale for computer literacy. In contrast, the CAI group had a higher mean on the cognitive subscale, with the sharpest differences appearing on the

applications and impact components of the cognitive subscale for computer literacy. Both groups showed positive attitudes towards computers, but neither group was considered computer literate in the cognitive domain.

Results of an ANOVA between the Control, CAI, and Programming groups showed significant differences between groups on both the affective and cognitive subscales. Newman-Keuls post-hoc analysis indicated that the Programming and CAI groups scored significantly higher than the Control group on the affective subscale, while the CAI group significantly higher on the cognitive subscale.

5. Interpretations

The investigators concluded that both the CAI and Programming treatments were effective in improving computer literacy in the affective domain. Only the CAI treatment was effective in improving computer literacy in the cognitive domain, but neither treatment resulted in an adequate level of computer literacy in this domain. The particular ineffectiveness of the Programming treatment in the cognitive domain of computer literacy is noted as puzzling. Suggestions are offered as to the possible reasons for this and the relative effectiveness of the CAI treatment as compared to Programming. Finally, it is noted that the results may be a result of the particular selection of items on the cognitive subscale of the MCLAA.

In concluding, the investigators suggest that this study supports explicit rather than incidental instruction on the cognitive aspects of computer literacy -- how computers work, what they can do, their use in society. In particular, the weakness of programming instruction in furthering these goals is compared with outcomes related to instructing and controlling the computer.

Abstrator's Comments

It appears that this report is based upon a reanalysis of data from the two prior studies referenced. With particular regard to the post-test, the fact that Control and CAI groups took an 83-item test whereas the Programming group took only the 56-item test might influence the relative scores -- even though only the common items were analyzed.

The treatment in the Programming group was, in contrast to the other groups, much less structured and, despite disclaimers, appears more prone to the influence of an instructor. Thus, the findings with respect to this treatment need to be considered as preliminary and subject to further study and experimentation. In fact, the entire study should be considered as exploratory and suggestive of avenues for further research to confirm and/or extend the results.

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Cooper, Lynn A. and Shepard, Roger N. TURNING SOMETHING OVER IN THE MIND. Scientific American 251: 106-107, 110-114; December 1984.

Abstract and comments prepared for I.M.E. by DAIYO SAWADA, University of Alberta.

1. Purpose

The purpose of the authors was to present a synthesis of their recent work into the relationship between mental representation and the physical processes thus represented. Their presentation provides an answer to the question: Can "the mind model physical processes, subjecting them to the geometric constraints that hold in the external world?" (p. 106).

They begin their account with an everyday example concerning a German Shepherd dog who retrieved a long stick that had been thrown over a fence that had one vertical board missing. The problem was solved with relative speed by the dog, who, rather than jarring his skull to his tailbone while bounding through the fence, stopped strategically before the impending collision, "paused and rotated its head 90 degrees" (p. 106), thus averting a collision. Thus, dogs (at least this one) seem to have a kind of spatial imagination that allow them to handle rotations mentally.

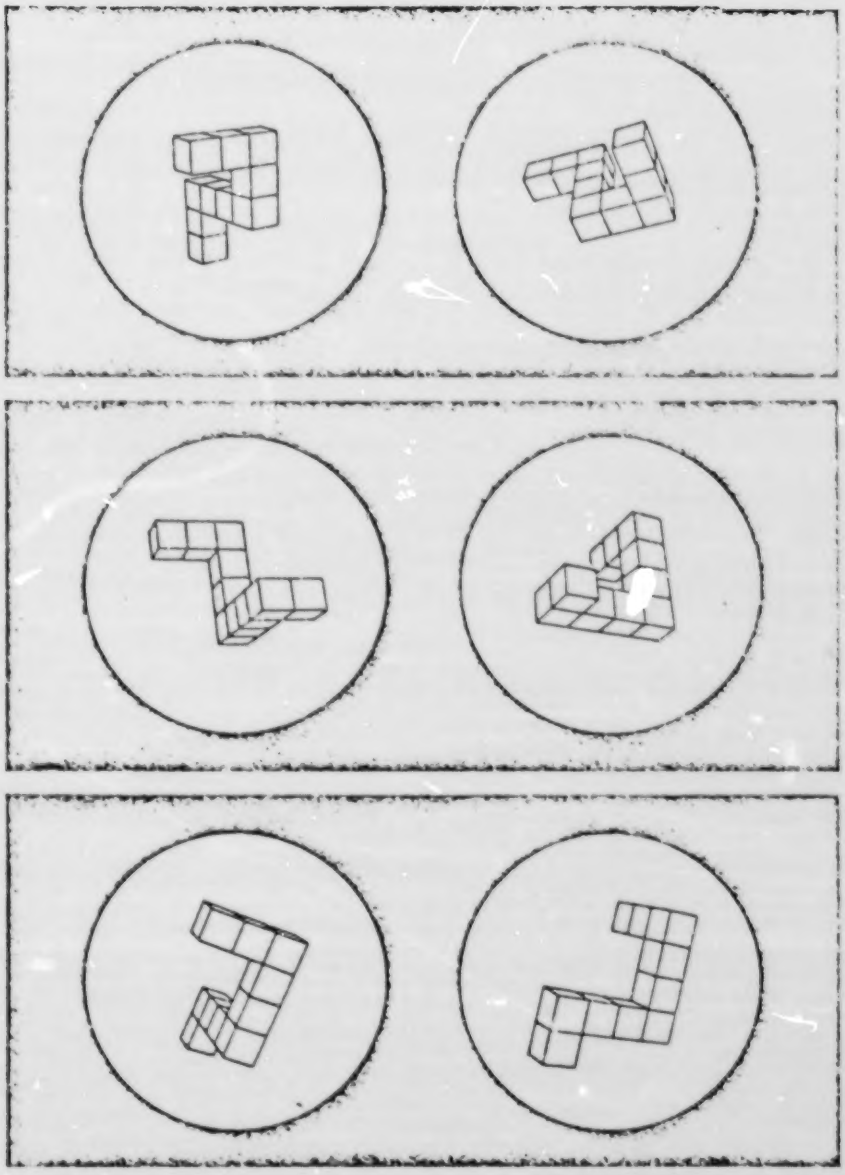
The authors then give a brief historical synopsis of views of spatial imagination beginning with the introspective accounts of well-known scientists (e.g., Kekule's dream-related image of the benzene ring structure). Such evidence, however, was found wanting: "subjective and qualitative assessments, even those made by scientists, cannot substitute for an objective and quantitative understanding" (p. 106). Cooper and Shepard then go on to indicate that the Behaviorist solution to the subjective nature of introspection was to banish reference to mentalistic terms such as consciousness or imagining. Their own research has been "to probe the kind of mental process the behaviorists ignored in a way that meets the behaviorists' demand for objectivity and quantitative

data. . . [In the research reported] each experimental trial was objective in the sense that the subject's response to a stimulus was either objectively correct or incorrect, and quantitative in the sense that the variable of interest was the time it took the subject to respond correctly" (p. 107).

Cooper and Shepard then went on to describe three experiments that together provide the evidence to support the answer they give to the question presented at the beginning of this abstract. Even though all three experiments are compelling and lead progressively to the validity of their position, I have chosen to devote the space available in this abstract to a more intense description of the first study so that the nature and substance of their rather ingenious approach can be appreciated.

2. The First Experiment

The eight subjects (young adults) compared computer-generated two-dimensional perspective line drawings, presented in pairs, of three-dimensional objects composed of 10 cubical blocks joined together face to face to form an arm-like structure with three bends [see figure 1]. Certain of the pairs were congruent but oriented differently in space; other pairs were not congruent but differed only by a reflection (enantiomorphic pairs).



PERSPECTIVE VIEWS displayed in pairs to the subjects of the authors' first experiment differed in three ways. In the first case (*top*) the drawings showed identical objects in positions that differed by a rotation within the plane of the picture. In the second (*middle*) the orientations portrayed differed by a rotation in depth. Subjects determined the identity of the objects in pairs of both types equally quickly, which suggests that in both cases they imagined the objects as three-dimensional solids rotating in space in order to compare them. A third kind of drawing pair used in the trials depicted enantiomorphic, or mirror-image, shapes (*bottom*).

Figure 1

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Each subject looked into a darkened tachistoscope into which E inserted a pair of drawings. The switch that was thrown to illuminate the drawings simultaneously started a timer. The subject compared the two drawings to decide as quickly as possible whether the objects portrayed were either the same or different, and registered the decision by pulling one of two levers for same or different.

3. Results

Introspectively:

> Subjects reported they could compare the shapes only by imagining one of the two objects rotated into the same orientation as the other and then checking for a match. Typically they said that they imagined the object on the left turned until its top arm paralleled the corresponding arm of the righthand object, they then mentally checked to see whether the extension at the other end of the object projected in the same direction as the analogous section of the companion structure (p. 110).

While informative, such verbal accounts are neither objective nor quantitative. The reaction times, however, are both objective and quantitative. Hypothetically, the length of time that it took to carry out the mental transformations in the mind should depend on the degree of transformation required to bring the lefthand object into an orientation parallel to the target object: the larger the rotation, the longer the reaction time. This is precisely what the data showed when graphed. As seen in figure 2:

> The times increased as a linear function of the angular difference between the orientations portrayed. When like objects were displayed in the same orientation, subjects took about a second to detect identity; with increasing angular difference the response times rose steadily (p. 110).

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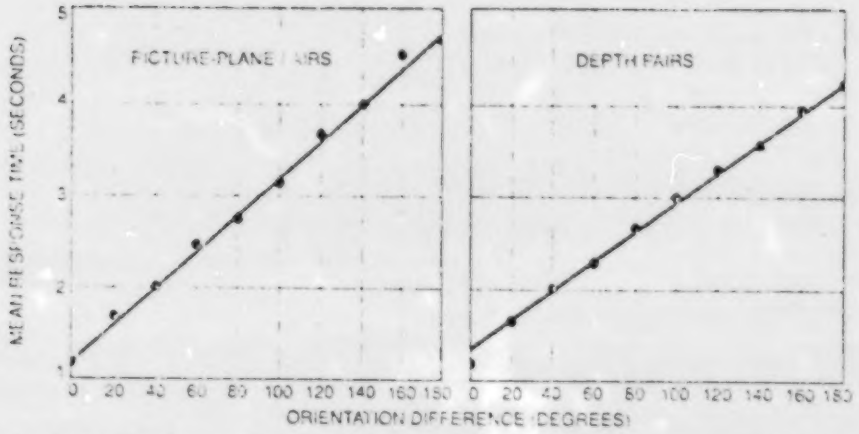

MEAN RESPONSE TIMES increased in direct proportion to the angular difference in orientation of identical objects presented for comparison in pairs of drawings. The linear relation suggested that a process of mental rotation underlies such comparisons. The slope of the function, from which the rate of the imagined rotation can be estimated, was no greater for orientations differing by a rotation in depth than for those differing by a rotation in the picture plane.

Figure 2

Could there be an alternate hypothesis that would explain the data as well as does the "mental rotation" explanation? The authors maintain that no other alternate explanation would serve as well. "Excluded, for example, is the possibility that subjects analyzed each drawing of a pair separately to reduce its structure to a code of some kind and then compared the coded descriptions" (p. 111). They then give an example code based on number and show how it would be inadequate.

Careful analysis of the drawings also revealed that what the subjects transformed (rotated) mentally was the three-dimensional object represented, not the two-dimensional features of the drawing. A perusal of the objects shown in figure 1 indicates that the top pair involves a rotation within the plane of the picture (two-dimensional representation), while the middle pair involves a rotation in depth. Nevertheless, subjects determined the identity of the objects equally quickly. As the authors conclude, "The rate of imagined rotation was as fast when the transformation portrayed involved three dimensions as it was when the rotation appeared to take place in two dimensions" (p. 111). A comparison of the graphs shown in figure 2 support this conclusion.

At the close of the first experiment, the authors concluded that:

> The progressive and spatial nature of imagined rotations, established in the first experiment, suggests that the process is analogous to transformations in the physical world. It is tempting to view the imagined roation as the internal simulation of an external rotation. Such a description, however, would be justified only if we could demonstrate that the internal process passes through intermediate states corresponding to the intermediate orientations of a physical object rotating in the external world (p. 111).

The second and third experiments go on to substantiate the correspondence of the intermediate states of the mental and physical rotations. Both experiments are models of careful and accurate experimental design; a brief summary would not do justice to the intricate operationalization that characterizes these experiments. I chose therefore to go directly to the conclusions, and recommend that readers in search of the details of the evidential support consult the original paper. Cooper and Shepard conclude their paper as follows:

> It may not be premature to propose that spatial imagination has evolved as a reflection of the physics and geometry of the external world. The rules that govern structures and motions in the physical world may, over evolutionary history, have been incorporated into human perceptual machinery, giving rise to demonstrable correspondences between mental imagery and its physical analogues. We begin to discern here a mental mechanics as precise and elegant as the innate schematism posited by Chomsky as the foundation of language (p. 114).

Abstractor's Comments

I believe this study is most significant for mathematics education in three areas: (1) pedagogy of mathematics, (2) research in mathematics education, and (3) epistemological concerns linking mathematics to the "external world".

Pedagogical Concerns

The pedogogy of mathematics has long assumed that experience with handling concrete manipulative aids helped the child to acquire mental constructs that somehow represented such actions. The findings reported in this paper suggest that physical action on objects indeed is directly simulated in the mind as a mental process. Although further study will be required documenting and substantiating the generalizability of the claims made in this paper, teachers can legitimately have greater confidence that children's play with appropriate concrete and ikonic aids will lead to mental operations that correspond to the physical transformations. Furthermore, the current inclusion of topics of Motion Geometry in school mathematics has support from this report in that the transformations of motion geometry are seen here as the transformations that are internalized.

Research Concerns

Cooper and Shepard place a great deal of credence in the "objective" and "quantitative" evidence they supply within the rather ingenious designs of their experiments in substantiating earlier belief based largely on everyday experience and qualitative research such as Piaget's (although they do not refer to Piaget). Between the lines of this report is the assumption that research of the clinical sort is really insufficient to build a science of cognition. Although much of what they conclude from their data could be predicted from Piaget's theory in

which action in the "real" world becomes internalized as mental operations in the mind, still there is a difference between verbal reports on the one hand and "objective and quantitative" data on the other. The questions I am left with are these:

(1) Can statements of conscious knowledge given as verbal reports by subjects about their performance (as in clinical research) be taken at face value as valid sources of information to substantiate theories?

(2) Are "objective" and "quantitative" data superior in some fundamental sense to verbal reports given by subjects about their performance?

(3) Can these two kinds of information be combined in complementary fashion to give two faces of the same phenomenon?

My personal inclination is to pursue the third question.

Epistemological Concerns

Imbedded in this report is the assumption that "reality" is somehow "out there" and the job of research is to develop an account that describes how the reality out there corresponds to constructs in the mind. Such a stance allows for a sense of "objectivity" to characterize the research. Despite this, I get an uncomfortable sense of illusion; that research of this kind unknowingly projects "out there" what it is ultimately going to find out there, and then suggests that "objective" evidence has thus been provided for the conclusions. This report finds that the mind has "mental representations of rotational transformation" that correspond to the physical rotations of objects in external reality. Could we not say as well that the report has designed a special experimental reality that embodies the currently dominant conception of space,

and has "discovered" that the experimental construction was indeed a valid embodiment of our current conception of space? In other words, is space objectively Euclidean? Or do we find it to be "objectively" and "quantitatively" Euclidean simply because we (and the subjects in the study) believe that it is Euclidean? How subjective is this objective space that we assume to be Euclidean, and then project it on nature? Jones (1982), a physicist at the University of Minnesota, says it this way:

> Whose reason do we see when we peer out there into space. . . The purely mathematical view of modern science, which has replaced the mechanistic view of Newton, is forcing us ever closer to the untenable position that the physical world is our own projection (p. 125).

I am suggesting that the "objectivity" that Cooper and Shepard claim is a characteristic of their data is as illusory as the "objectivity" of an Escher drawing. In other words, if an experiment were done which involved perspective drawings such as created by Escher, then the "reality" of the Escher space would indeed NOT correspond to the mental representations of young adults. In this sense, there really is nothing terribly "objective" about research of the kind reported in this paper.

Nevertheless, this research is indeed most ingenious and insightful concerning the operations of the mind. I think it is the creative analytical precision of Cooper and Shepard that lends credence to their work rather than its ostensive objectivity. Good research of whatever kind is good, not because it is of a particular kind, but because the researchers show creativity and accuracy in dealing with their phenomenon. Such is the case here.

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Darch, Craig; Carnine, Doug; and Gersten, Russell. EXPLICIT INSTRUCTION IN MATHEMATICS PROBLEM SOLVING. Journal of Educational Research 77: 351-359; July/August 1984.

Abstract and comments prepared for I.M.E. by ERIC W. HART, Maharishi International University, and HAROLD L. SCHOEN, The University of Iowa.

1. Purpose

The purpose of this study was to examine the effectiveness of an explicit method of teaching story problems to skill-deficient fourth graders, as compared to the method used in the four state-adopted arithmetic texts in Oregon. The study also examined the effects of providing extra practice for material not mastered.

2. Rationale

The instructional sequence used in this study was based on principles from Engelman and Carnine (1982). Marcucci's 1980 meta-analysis showing that "guided discovery" methods for teaching problem solving are rarely effective is referenced to support the investigators' study of the effectiveness of an explicit, sequential method. Further, their explicit method is stated to have "many common features with other direct instruction (Rosenshire, 1983) or active teaching (Good, Grouws, and Ebmeir, 1983) approaches" (p. 358). There are some differences, however, and among these is the explicit method's provision of detailed correction procedures--procedures suggested by Silbert, Carnine, and Stein (1981).

3. Research Design and Procedures

Subjects were 73 fourth graders, from six classrooms in a school district in the Northwest, who demonstrated skill deficiency in problem solving and yet had basic computational ability (as shown by two tests administered by the investigators). They were randomly assigned to one

of four experimental groups: (1) explicit method/fixed amount of practice; (2) explicit/additional practice; (3) basal instruction developed from the four texts/fixed practice; (4) basal instruction/additional practice. Another dimension of the analysis was posttest-maintenance test.

Students were taught in groups of two to four. The teachers involved were four graduate students in special education who received systematic training and used detailed, semi-scripted teaching manuals. Identical multiplication and division story problems were taught to all students.

The fixed amount of practice consisted of eleven 30-minute lessons, while the additional practice included up to eight additional lessons (as needed, based upon performance on tests administered on three different days). Additional practice sessions were composed of both extra instruction and extra practice problems.

The explicit translation strategy comprised four components: (1) The students were taught the rule, "If you use the same number again and again, you multiply." This rule was first used with concrete models and then with actual word problems. They were told to look for the word "each" or "every" as a signal for multiplication. (2) Next, students learned the rule, "The big number tells how many there are in all. If the big number is not given, the problem is a multiplication problem. If the big number is given, it is a division problem." (3) Finally, students were taught to discriminate among all types of problems by asking the questions, "Does the story deal with the same number again and again?" and, "Does the story give the big number?" (4) Systematic corrective procedures were an important part of the method.

The basal instruction method was a composite from the four state-adopted tests. It consisted of three components: (1) "Discussion designed to increase student involvement and motivation" (p. 354).

(2) A four step system for solving problems: (a) place numbers here; (b) identify correct operation; (c) write and complete the number sentence; (d) place your answer here. (3) Corrective procedures were used in this method but they were much less systematic than those in the explicit strategy.

The criterion measures were a posttest, a maintenance test given 10 school days after the posttest, and a "satisfaction questionnaire." The posttest and maintenance test were parallel instruments, each containing 11 multiplication story problems, 9 division problems, and 6 addition or subtraction problems to serve as distractors.

4. Findings

A 2x2x2 ANOVA was used to analyze the data. Similar results were found whether students or instructional groups were used as the unit of analysis:

(1) On the posttest, a significant effect was found for type of instruction, with the explicit method preferred. There was no effect for the practice variable. Of the explicit students, 86% achieved a level of acceptable performance (80% correct), while only 24% of the basal students performed acceptably.

(2) On the maintenance test, there was a significant interaction between the instruction variable and the practice variable. The explicit/additional practice group performed better than either of the basal groups; there was no difference between the explicit/fixed practice group and any of the other three groups. Providing one or two extra lessons when needed "appeared to have an impact on the explicit students" (p. 357).

(3) The student questionnaire showed that students liked the explicit method better and used it more often in their regular mathematics classes.

(4) The explicit/additional practice students averaged 1.7 extra lessions, while the basal/additional practice students averaged 6.7 extra lessions. Nevertheless, the explicit students still scored well above the basal students.

5. Interpretations

"The results of this study do clearly demonstrate the benefit of analyzing an instructional domain, such as translation of word problems, in fine detail to determine the essential discriminations and strategies that students must learn to perform adequately" (p. 357). The investigators state that the basal textbooks do not use such an analysis, and that, in fact, the guided discovery and discussion methods which they do not use "appear to be questionable procedures for use with lower performing students" (p. 358). They state further that, "The results of this study support the position that a program constructed to teach prerequisite skills in a sequential manner and, more importantly, explicitly model and teach each step in the translation process is significantly more effective than approaches advocated in teachers' guides to currently used basal series" (p. 358). They point out that the students in the study seem to agree, as evidenced by such comments on the questionnaire as, "I liked using rules to work problems" (p. 357).

Abstractors' Comments

This study is a well-designed study in the behaviorist tradition. The question addressed is of practical significance to elementary school teachers, that is, how can one raise the story problem test scores of fourth graders who are having difficulty? The answer these

researchers propose involves analyzing the instructional domain of multiplication and division story problems, developing very explicit heuristics (i.e., rules) based on that analysis, and then directly teaching students to use those rules to solve the problems.

On tests containing problems to which the rules apply, students who were taught the rules scored significantly higher than students who were taught using the standard textbook approach.

No processes were examined in this study. However, since other research has shown that good "problem solvers" in elementary school use contextual rules similar to those in this study, such as cue words and comparison of number sizes, it seems safe to assume that the experimental subjects scored better because they were indeed applying the rules they were taught.

While the positive results of this study are encouraging, it seems appropriate to consider them in terms of the goals for teaching story problems in elementary schools as well as the methods used to attain those goals. Even though there is not consensus on any one goal for teaching story problems, most mathematics educators would include at least one of the following three as essential: (a) to develop general problem solving ability; (b) to develop better understanding of mathematical concepts and operations by applying them in various settings; and (c) to develop the ability to apply mathematics in useful real-world situations. As far as attaining these goals, recent research supports the teaching of heuristics specific to the given domain of problems. However, this study carries the degree of specificity beyond what is normally done, and thus raises the following questions:

(1) Are increased test scores due to the use of specific rules in any narrowly-defined problem domain (e.g., the explicit rules in the domain of multiplication

and division story problems in this study) necessarily indicative of progress toward the above-mentioned goals? Or is the increase due to an improvement in test-wiseness?

(2) What degree of specificity in teaching heuristics will ensure success in the target problem domain and yet still provide positive transfer to other domains? In light of this question, is the explicit strategy in this study too specific?

(3) Is it possible (desirable) to develop rules, such as was done in this study, for all problem-solving domains?

(4) Will an over-emphasis on explicit strategies, such as the strategy in this study, lead students to the mistaken belief that mathematics is no more than a set of fragmented, rule-bound procedures?

In closing, we found this study to be thoughtfully designed and carefully executed. Our only quarrel is with the philosophy of teaching word problems that is implied.

Enochs, L. G. and Gabel, D. L. PRESERVICE ELEMENTARY TEACHERS' CONCEPTIONS OF VOLUME. School Science and Mathematics 84: 670-680; December 1984.

Abstract and comments prepared for I.M.E. by MICHAEL T. BATTISTA, Kent State University.

1. Purpose

The purpose of this study was to identify preservice elementary teachers' "misconceptions of volume and surface area through written observation and interviews" (p. 671).

2. Rationale

Many preservice elementary teachers have difficulty in finding the volume and surface area of objects such as rectangular solids. According to the authors, little research on preservice elementary teachers' learning of these concepts has been conducted. Thus, in order to improve instruction on volume and surface area, this study attempted to identify such students' misconceptions of these important concepts. Surface area was included in this study of volume "because many students do not distinguish between volume and surface area. A thorough understanding of volume would include this distinction" (p. 671).

3. Research Design and Procedures

In order to probe students' misconceptions, a Surface Area/Volume Misconception Inventory (SAVMI) was developed and administered to 125 preservice elementary teachers (98% female) enrolled in a freshman basic-skills science course. The SAVMI was given at the beginning of the semester. Six 13-item forms of the SAVMI were developed -- one for volume and one for surface area for a rectangular solid, a cylinder, and a rectangular solid with a small rectangular hole in it. Each preservice elementary teacher was given two forms of the SAVMI, one on volume and one

on surface area. Each of the thirteen items on a SAVMI asked students whether the volume or surface area of the figure drawn on the form could be calculated using a given method. Students were to respond "yes," "no," or "uncertain." For example, when students were asked about finding the volume of a rectangular box, some methods that were presented to them were: "Multiply the area of the base x height. Use the formula 6(LxWxH). Count the number of cubes 1 inch on each side that would fill the object."

In addition to administering the SAVMI, the investigators interviewed eight volunteer preservice elementary teachers who scored zero on a test of volume and surface area. The interviews took place following a lesson on surface area and volume. The students were asked to think aloud as they attempted to determine the volume and surface area of objects depicted by "solid three dimensional models, hollow models, and drawings of each" (p 674).

4. Findings

The authors found that the preservice elementary teachers had "a statistically significant better understanding of volume than surface area" (p. 675). However, the means were quite low for both volume and surface area (7.74 and 6.70 out of 13, respectively), and the authors concluded that preservice elementary teachers "do not really understand either of these concepts" (p. 675). Students had greater difficulty with the SAVMIs for the volume of cylinders and solids with holes than for rectangular solids, but this trend was not statistically significant. No such trend was evidenced for surface area.

Of the five correct ways listed on the SAVMI for finding the volume of a rectangular solid, 77% of the students were certain that the volume could be obtained by multiplying length times width times height, but only 44% were certain that volume was equal to the area of the base times the height, and only 58% were certain that the volume could be found by

counting the number of cubes that would fill the solid. Students were even less certain about how to determine the surface area of a rectangular solid. For instance, only 34% knew that the surface area could be determined by counting the number of unit squares on the outside of the box.

5. Interpretations

The authors interpret the data as indicating that "a large percentage of elementary education majors do not understand the concept of volume and are unable to distinguish volume from surface area. . . . They were found to solve problems using a 'memorizing mode' rather than basing their answers on the concept itself" (p. 679). The authors suggest that instruction for these students that employs a "hands-on approach without mention of formulas until the end of the instruction would be more effective in teaching" these concepts (p. 679). They conclude that "If elementary teachers are expected to teach volume in the schools they must first understand the concept themselves. If they do not, volume will be taught as a formula to be memorized and applied, rather than as an entity in itself" (p. 677).

Abstractor's Comments

To anybody who has taught the concepts of volume and surface area to preservice elementary teachers, the results of this study come as no surprise. The value of the study is that it serves to document some of the misconceptions students have about these extemely important concepts. While I found the results of the study very interesting, there were several additional areas that I would have liked the authors to address. (In fairness to the authors, it should be mentioned that probably not all of these areas could have been covered in one article because of space limitations. Perhaps some of them could be addressed in future studies.)

First, I felt that the author's interviews could have delved a little deeper into some of the misconceptions. For instance, in summarizing the interview findings, the authors state that "It was evident that these students did not understand the concepts of volume and surface area (after two hours of instruction) because they used units interchangeably for length, volume, and area (cm, cm^3, cm^2)" (p. 677). Although I, too, believe that one reason that students so often give the wrong units for answers to problems involving volume and area is their lack of understanding of these concepts, I do not think that we can assume that giving the wrong units implies such misunderstanding. It could be simple carelessness. It could be a misunderstanding of the use of units, not of volume or surface area. This is one question that more in-depth interviews could have helped us better understand.

Second, although the authors' introductory remarks discuss the importance of spatial visualization skills in mathematics learning, no mention of it is made later on in the study. I would like to have seen the authors measure spatial visualization and at least attempt to relate it to students' misconceptions about volume and surface area. For instance, is spatial visualization positively correlated with preservice elementary teachers' understanding or computation of volume or surface area?

Third, Table 4 of the article presents percentages of students who used various correct and incorrect methods for determining the volume and surface area of a rectangular solid. I believe further analysis of this table would have been useful, especially since several of the table's entries were startling. For instance, only 9% of the students indicated that the formula 6(LxWxH) was invalid for determining volume. Since 77% identified LxWxH as a proper formula for finding volume, at least 68% of the students thought or were uncertain that both formulas could be used for volume. (This result was so surprising that I called one of the authors to be sure that I was correctly interpreting the

table.) Less dramatically, 58% of the students thought that the volume of the solid could be determined by counting the number of cubes that would fit inside, 58% thought that it could be determined by measuring the amount of water that would fill it, and 77% said that it could be determined by multiplying the length by the width by the height. It would be interesting to know if all 58% of the students who evidenced a conceptual understanding of volume knew the standard volume formula. Even more interesting would be to know how many of the 58% of students who had a conceptual understanding failed to recognize the various incorrect methods for finding volume. Or, how many of the students who knew the standard volume formula but did not evidence a conceptual understanding of volume chose the various incorrect procedures for finding volume? A similar analysis could have been done for surface area. Furthermore, the discussion of errors was restricted to rectangular solids. Were similar results obtained for cylinders and solids with holes?

Table 4 also showed that only 14% of the students indicated that the volume of a rectangular solid could not be found by counting the number of unit squares on its surface; so 86% either thought that this method could be used to find the volume or were uncertain about whether it could be used (another startling finding). Fifty-eight percent of the students indicated that the surface area of the solid could not be found by counting the number of unit cubes that would fit in the solid; so 44% thought that this method was valid or were uncertain. Thus, the data did seem to support the authors' contention that preservice elementary teachers tend to confuse volume and surface area. But it would be interesting to further investigate the relationship between student responses. For instance, were the 58% of the students who recognized that counting the cubes that fit in a solid is not a valid method for determining surface area the same 58% who said that this method was valid for determining volume? Furthermore, only 34% of the students indicated that multiplying LxWxH was an invalid method for determining

the surface area of a rectangular solid. I wonder if the 66% of students who failed to recognize the invalidity of this method had such a weak conception of surface area that, when asked about methods for its determination, retrieved information from the more well-developed volume "frame" from memory rather than the weak or nonexistent surface area "frame." This would parallel the common error that elementary children make when for the problem 8x8 they answer 16. (See Davis (1984) for his discussion of frames and binary reversions.)

Finally, the authors did not relate their findings to other research in this area. One piece of research that is especially relevant is the National Assessment of Educational Progress. Indeed, Carpenter et al. (1980) reported that only 39% of all 17-year olds (57% of those who had taken a full year of geometry) could find the volume of a rectangular solid. They also reported that 18% of all 17-year olds added the three dimensions given for the solid to get the volume. This particular error, however, was not listed in the SAVMI for the volume of a rectangular solid. Instead, the authors listed three other incorrect formulas involving these three dimensions as methods for finding volume. The percent of preservice elementary teachers who recognized that the volume of a rectangular solid could not be calculated using these methods ranged from 9 to 21%. Thus, one wonders about the underlying reasons for the majority of students failing to recognize that these formulas are not valid for calculating volume. Is it a misunderstanding of the concept of volume, an incomplete memory of a formula that involves these three dimensions, or an inability to physically interpret the formulas?

In order to corroborate the authors' results, I decided to examine students' performance on some exam items from a geometry course for preservice elementary teachers that I teach. I was able to collect data on two sections of the course (n=71) taught in two different semesters. Of course the exams were given to students after they had had instruction over the topics of volume and surface area. There were two items that asked students to find the volume and surface area of a rectangular solid. On the item that was presented verbally, 83% of the

students correctly calculated the volume and 76% the surface area. On the item that was presented pictorially, 86% of the students correctly calculated the volume and 72% the surface area. The authors reported that 67% and 26% of their students correctly calculated the volume and surface area of such a figure, respectively. Thus, even after recent instruction, significant numbers of preservice elementary teachers are unable to calculate the volume and surface area of a rectangular solid, and surface area remains more difficult to calculate than volume.

Also included on these two exams was the following item that was designed to detect if students were confusing the concepts of surface area and volume.

> A small girl is playing with two boxes -- box A and box B. She has found that she has to stick 15 identical postage stamps onto box A in order to completely cover it, whereas box B requires 16 stamps to be covered. She has also found that it takes 18 sugar cubes to fill box A and 16 to fill box B. What can we conclude about the boxes?
>
> a) The surface area of box A is less than the surface area of box B, and the volume of box A is greater than the volume of box B.
>
> b) The surface area of box A is greater than the surface area of box B, and the volume of box A is less than the volume of box B.

c) Box A will fit into box B.

d) The length plus the width plus the height of box A is greater than the length plus the width plus the height of box B.

e) none of the above.

Thirteen percent of the students missed this item. (Out of 71 students, 62 chose (a), 2 chose (b), 3 chose (d), and 4 chose (e).) So, after instruction in which concrete materials were employed and the difference between volume and surface area was explicitly discussed, only 3% of the preservice elementary teachers confused volume with surface area. Apparently, however, other misconceptions about these concepts persisted for another 10% of the students.

Finally, I found that 13% of the students used the wrong units for volume on at least two of three items; 16% used the wrong units for surface area. Thus, students do not always mislabel the units merely out of carelessness, so this phenomenon should be investigated more carefully.

In conclusion, the authors have conducted an interesting study that provides us with some useful results about preservice elementary teachers' misconceptions of volume and surface area. More research is needed, however, and at a deeper level of analysis, before we can truly understand the nature of these misconceptions.

References

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Minato, Saburoh; Yanase, Shyoichi. ON THE RELATIONSHIP BETWEEN STUDENTS' ATTITUDES TOWARDS SCHOOL MATHEMATICS AND THEIR LEVELS OF INTELLIGENCE. Educational Studies in Mathematics 15: 313-320; August 1984.

Abstract and comments prepared for I.M.E. by CHARLEEN M. DERIDDER, Knox County Schools, Tennessee.

1. Purpose

The purpose of this study was to replicate a previous study which examined the effect of student attitudes toward school mathematics on achievement scores in mathematics. The results of the first study indicated that there was a differential effect of attitudes on achievement according to student ability level. This study was conducted in an attempt to confirm the previous results.

2. Rationale

It was stated in this study that previous research has shown a modest correlation between achievement and attitude in school mathematics and that Burek (1975) had found more cases where students' attitudes caused mathematical cognitive achievement than the reverse. Although no pattern of causal relationships could be identified as a significant result in Burek's work, this study was based on the assumption that attitude is a causal factor with respect to the level of achievement in mathematics.

3. Research Design and Procedures

A total of 808 eighth-grade students from three Japanese schools, different from those in the first study, were tested using the following instruments. There were two attitudinal tests; the MSD, a semantic differential instrument developed by Minato (1983) in Japan, to measure attitudes towards school mathematics using 17 bipolar adjective pairs

each with a seven-point continuum, and the FA, developed by Minato and others, a Likert-type instrument to assess student attitude with respect to its being "favorable to school mathematics." (No complete names were offered for the MSD and the FA). There were summative tests, written by mathematics teachers in each of the schools, to assess achievement concerning numbers, linear equations, and inequalities. These tests were administered during the same month to all subjects.

The students were grouped within each school according to ability level based on an intelligence test which, according to the study, is used extensively in Japan. This testing preceded the attitude and summative tests by three months.

The study posed two hypotheses based on two different groupings of the subjects. Hypothesis 1 is that there are inequalities on the regression coefficients (b) of three different groups, with the b of the low intelligence group > the b of the middle intelligence group > the b of the high intelligence group. Hypothesis 2, using two groups larger in number than the three and separated by low and high intelligence, is that the b of the low group is > the b of the high group.

Regression coefficients of summative (achievement) test scores on the two attitudinal tests were obtained and the authors stated that the results matched their hypotheses. However, there was no indication of significance at this point.

The authors then tested the statistical null hypothesis using an analysis of covariance with a regression coefficient, which represents the rate of increment of achievement test score in relation to the increment of attitude scores, with attitude as the causal factor.

4. Findings

As was stated above, there was no indication of statistical significance with respect to the regression coefficients of the summative test scores on the two attitude tests.

The use of ANCOVA for hypotheses 1 and 2 resulted in the authors' determining a single regression coefficient for each school with respect to the achievement test scores on each of the attitude tests, but with no indication of group comparisons within a given school or across the three schools. However, the study states that using the technique of ANCOVA, hypothesis 1 was accepted (meaning the rejection of the null hypothesis) at the .05 level for only one school, and that hypothesis 2 was accepted for all schools at least at the .05 level.

5. Interpretations

According to the study, it can be interpreted that, if attitude toward mathematics affects the learning of it, then the effect is different for different levels of students' intelligence, and that the attitude of low intelligence students is more important in that it affects more in magnitude than that of high intelligence students.

The study does note that because there is little evidence that the causality of attitude on achievement is true, and that this study is based on that assumption, no conclusive result has been obtained.

Abstractor's Comments

There are several questions raised as one examines this study, due to the vagueness and lack of clarification of its report. These questions or concerns are:

a. Were the teacher-made summative (achievement) tests different for each school? The study implied this. Was there any attempt to measure the validity or reliability of these tests?

b. There was no mention of the validity or reliability of the other test instruments used.

c. What method was used to determine correlation coefficients? The data appeared to suggest the need for a non-parametric method, but the study did not clarify that question.

d. The use of analysis of covariance is meant to adjust for a variable which might affect the dependent variable. Such a covariate was never identified. It appeared that it might be measures of intelligence, but that was used to identify the groups initially. There was no random sampling of groups, which seriously questions the use of inferential statistical methods.

e. The ANCOVA results indicated one regression coefficient for each school. This meant that the identity of the high, middle, and low or high and low intelligence groups was not contained. Consequently, the results no longer appeared to correspond to the hypotheses.

f. The study was based on the assumption, not at all well substantiated, that attitude is a causal factor in levels of achievement. It would appear that some type of longitudinal study might be more appropriate to determine the causal or interactive relationships between attitude and achievement variables with respect to their correlations.

The authors of the study were reserved in their interpretation that they did not obtain any conclusive result. Their study, however, does provide a stimulus for future studies on attitudes toward school mathematics with special attention to the attitudes of the less able student.

Reference

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Muth, K. Denise. SOLVING ARITHMETIC WORD PROBLEMS: ROLE OF READING AND COMPUTATIONAL SKILLS. Journal of Educational Psychology 76: 205-210; April 1984.

Abstract and comments prepared for I.M.E. by DALE DROST, University of New Brunswick.

1. Purpose

The purpose of the study was to determine the relative importance of computational ability and reading ability to the solution of arithmetic word problems by sixth graders.

2. Rationale

The author cites the need "to identify the component abilities that contribute to the successful solution of arithmetic word problems" (p. 205). It is claimed that agreement exists that computational ability is essential for solving such problems, but no supporting research is presented. Studies with conflicting conclusions regarding the importance of reading ability are quoted; however, those which identify this factor as playing only a minor role in problem-solving ability are criticized with respect to design.

Examples are given to illustrate that in solving problems in the real world, a distinction must be made between relevant and extraneous information and often this information is embedded within complex textual formats. The presence of extraneous information is associated with the computational demand of a problem, whereas the syntactic complexity is associated with the reading demand.

3. Research Design and Procedures

Two hundred sixth graders were administered the Comprehensive Test of Basic Skills to obtain measures of reading ability and computational ability from the reading comprehension subtest and the arithmetic computation subtest, respectively.

Four similar 15-item arithmetic word problem tests were constructed using adaptions of problems from the National Assessment of Educational Progress. On one test the problems were written using simple syntax and no extraneous information; on a second, they had complex syntax and no extraneous information; on a third, simple syntax and extraneous information, and on a fourth test, complex syntax and extraneous information. When written with simple syntax the problem contained three short sentences, whereas with complex syntax these three sentences were combined into one longer sentence. The extraneous information included one item of numerical information not necessary for the solution of the problem. Each test was administered to 50 of the students in the sample. Test reliabilities were between 0.80 and 0.90.

For each student, three performances measures were determined: the total number of correct answers, the total number of problems set up correctly, and the total test-taking time measured in seconds.

The data were analyzed by determining means and standard deviations for the ability and performance variables and correlation coefficients between all variables. A regression analysis was used to determine the relative contribution of each ability variable and each format variable to each performance criterion.

4. Findings

a) From the Comprehensive Test of Basic Skills the reading grade-equivalent scores ranged from 1.4 to 11.9 with a mean of 6.29, while the corresponding scores for computation ranged from 1.0 to 11.9 with a mean of 6.31.

b) Overall, students correctly set up an average of 9.11 of the 15 problems and obtained 8.68 correct answers. Problems without extraneous information were set up correctly an average of 10.84 times with 10.25 correct solutions, and those with extraneous information were set up correctly 7.38 times with 7.11 correct answers. Means were not reported for simple and complex syntax with problem information not considered; however, there was little difference between the two types of syntax, with all means approximately nine out of 15.

c) Test-taking time ranged from 303 to 1860 seconds with a mean of 907.67 seconds.

d) From the correlational analysis, reading and computational ability were positively correlated. Both of these ability measures were positively correlated with correct answers and set-ups, and negatively correlated with test-taking time. The presence of extraneous information was negatively correlated with correct answers and set-ups, and positively correlated with test-taking time; however, syntactic complexity was not correlated significantly with any of the other variables.

e) From the regression analysis, reading ability, computational ability, and problem information all made significant contributions to the regression equation when either correct answers or correct set-ups were used as the dependent variable. When test-taking time was the dependent variable, computational ability did not add significantly to the contribution made by reading ability; however, the addition of problem information to the equation increased the variance accounted for significantly. Test-taking times were faster when extraneous information was absent.

5. Interpretations

It was concluded that both reading ability and computational ability contribute to success in solving arithmetic word problems. Also, the presence of extraneous information increased both the difficulty of problems and the time needed to complete them. It was suggested that teachers need to design activities to integrate basic reading and computation skills and to enhance word problems in more realistic contexts with extraneous information.

Abstractor's Comments

The author is to be commended for a well-designed study and a well-written report. The design of the research was clearly reported, as were the procedures employed and the conclusions reached.

Although the results of the study are not surprising, they are valuable to classroom teachers in that they confirm the beliefs of those closest to children. Most teachers would readily agree that both reading ability and computational ability are highly related to children's success in solving problems. They would also agree that including extraneous information in problems increases their difficulty. One result not so obvious is that the inclusion of this information increases the time students need to complete the problems.

One result with which teachers might not agree was with respect to syntactic complexity. In this study there was no difference between performance on problems written using simple syntax and those written using complex syntax. It was disappointing that the researcher did not discuss this result. Why were differences not found? Perhaps the "complex" sentence formed by combining three short sentences was still read as three short sentences. This is quite possible in the example included in the report, the only additional words being an "if" and an "and", together with a comma. In her rationale, the author refers to

complex syntax used in various types of documents. Perhaps these should be investigated to obtain a different definition of "realistic" complex syntax. Further research is necessary in this whole area.

The example problem presented in the report is a one-step problem involving addition of two whole numbers. For the large majority of grade six students, this problem would be considered very easy. Were the other problems on the test of a similar difficulty? Were there any two-step problems? Were the students required to engage in any significant problem-solving activity or were the problems simply a computational exercise surrounded by a few words? Given that approximately 60% of the students answered the questions correctly and that many of them were well below grade level in reading and/or computational ability, it appears the problems were quite easy. Would the results of the study have been similar if problems had been more challenging for the students?

As suggested above, students in the sample varied greatly in ability. When completing the test, some students took as little as five minutes of time to complete 15 problems. It would be interesting to investigate these factors further. Were there any differences across ability levels on the dependent variables? Although test-taking time correlated significantly with all other variables except syntactic structure, the correlations were relatively low, in the order of +0.20. Did some students who scored high take little time? There are several possible interactions between the variables in this study which should be studied in subsequent research.

In summary, the study was well designed and reported. The author might have discussed the results at greater length and made suggestions for further studies.

Seifert, Edward H. and Beck, John J., Jr. RELATIONSHIPS BETWEEN TASK TIME AND LEARNING GAINS IN SECONDARY SCHOOLS. Journal of Educational Research 78: 5-10; September/October 1984.

Abstract and comments prepared for I.M.E. by MORRIS LAI, University of Hawaii.

1. Purpose

The study addressed the following questions: (1) Do relationships exist between student achievement gains and selected time-related variables at the secondary level? (2) Do relationships exist between student attitude changes and selected time-related variables at the secondary level?

2. Rationale

Although many researchers have found time to be a crucial variable related to school and teaching effectiveness, most of the research has been at the elementary rather than at the secondary level. Despite the popularity of the search for ways to improve school effectiveness, there has not been much change in high schools.

3. Research Design and Procedures

Within each of five high schools, a 40-item multiple choice objectives-based pretest and an attitude toward mathematics pretest were administered to two sections of first-year algebra students. Six students, three male and three female, whose pretest scores were near the classroom mean, were identified for observation. Neither the teacher nor the students were aware of the identity of the selected students.

Over a 10-week span, 10 observations of student behavior were collected each class period observed for a total of 100 data entries per student for each of the following five variables: (1) setting (the general teaching strategy common to high school mathematics classrooms; for example, "lecture/discussion"); (2) objective (whether students were on or not on the daily objectives); (3) learner moves (specific observed behaviors of the student during the instructional sequence; for example, "engaged, spoken response"); (4) general moves (for example, "free time" or "non-academic instruction"); and (5) interruption (observable breaks in the instructional sequence).

Posttests were administered in December to all students. The analysis was conducted on only the six selected students per classroom. Scattergrams, product-moment correlations, and two-tailed t-tests were computed.

4. Findings

Students were on task about 54% of the time. There was a relatively low on-task rate at the beginning of the class period, a maximum rate 16.5 to 22 minutes after the class period began, and a relatively low on-task rate at the end of the typical period.

The researchers found a statistically significant (alpha = .05) positive correlation of 0.46 between achievement gain and use of the lecture/discussion method. A statistically significant negative correlation of −0.50 was found between achievement gain and amount of seatwork. Total engaged time (on task) correlated 0.45 with achievement gain, with most of the variance accounted for by the category of covert engagement. Negative correlations were found between achievement gain and variables such as the amount of time off task ($r = -0.35$) and the amount of time spent waiting for help ($r = -0.37$). The correlation of 0.29 between attitude change and achievement gain was not statistically significant at the .05 level.

5. Interpretations

Students spend more time on task when teachers are in direct control of instruction. Most students achieve very little when given homework to complete without direct supervision. Students appear to achieve optimally when they are listening and thinking, when the teachers challenge the students' intellect. Students waiting for help appear to lose interest in learning and move to an off-task mode. The amount of on-task time appears to be lower than that found in similar studies; however, secondary school instruction appears to be more susceptible to off-task influences. The findings indicate that the more time students spend on task the more they will learn.

Abstractor's Comments

In studying the relationship between task time and learning gains in secondary schools, the authors have addressed an important research area. By adapting procedures developed for the Beginning Teacher Evaluation Study, they used state-of-the-art observation methodology in the conduct of their study. It was difficult, however, to comprehend the article because the authors used questionable logic and nonstandard ways of reporting and displaying results.

Table 1, entitled "Correlation of Observed Events and Achievement Gain," gives the correlation coefficients under the heading "Achievement Gain." The next column consists of five rows of "5.27," which corresponds to the overall mean achievement gain. Table 4, entitled "Correlation of Math Attitude and Achievement Gain," lists under the heading "Attitude Change" the phrase "Math Attitude." Both of these examples represent confusing, unconventional usage.

Further undermining the comprehensibility of the article are two major flaws: (1) The authors label achievement gain as the independent variable and the classroom observation variables as the dependent

variables, and (2) they imply in their writing that correlation implies causation (but in the sense that achievement gain is now a dependent variable). For example, they write, "...the instructional setting 'seatwork' appears to produce a negative learning situation when on-task time is considered a positive learning environment." Simiarly they conclude, "This appears to indicate that most students achieve very little when given homework to complete without direct supervision." An alternate conclusion could be that students who are low achievers tend to be given homework to complete without direct supervision.

Other parts of the study I found problematic were: (1) the use of a test based on first-year algebra objectives even though a relatively brief October-December pretest-posttest interval was used; (2) statements that went beyond the scope of the reported research; for example, "In this study the ability to keep students engaged and on-task appeared to be closely aligned to the teacher's ability to change auditory moods and challenge the student's intellect"; and (3) the authors' unconventional if not erroneous reporting of statistical significance: "When achievement gains for individual classrooms were compared to minutes of on-task time per classroom, a strong relationship was found to exist ($p > 0.95$)."

Although I would not disagree with the authors' conclusion that the more time students spend on task the more they will learn, because of the numerous flaws in the study I remain skeptical about many of the other reported findings and conclusions.

Slavin, Robert E. and Karweit, Nancy L. MASTERY LEARNING AND STUDENT TEAMS: A FACTORIAL EXPERIMENT IN URBAN GENERAL MATHEMATICS CLASSES. American Educational Research Journal 21: 725-736; Winter 1984.

Abstract and comments prepared for I.M.E. by GEORGE W. BRIGHT, University of Houston.

1. Purpose

The purpose was "to investigate the separate and combined effects of the principal components of mastery learning and team learning on student mathematics achievement" (p. 727). It was hypothesized that the effects of mastery learning together with team learning might be greater than the sum of the effects that might be predicted for the two treatments separately.

2. Rationale

The study arose from Slavin's (1984) analysis of mastery learning, which postulated that effective instruction must adequately deal with four alterable aspects of Carroll's (1963) model of instruction: (a) instruction appropriate to students' understanding, (b) incentives for learning, (c) time, and (d) quality of instruction. Recent research on mastery learning, with time among groups held constant, has tended to show little benefit for that model as a whole. Research on team learning has consistently shown positive learning effects. Thus, the argument for this research is that individual components of mastery learning must be studied in more depth in order to understand how the mastery learning model relates to instructional processes. Too, Slavin argues that the components may reinforce each other rather than being merely additive. Verifying this hypothesis would have great potential impact on the design of instruction.

3. Research Design and Procedures

Subjects were ninth-grade general mathematics students in inner-city Philadelphia. Of an initial sample of 1092 students in 16 schools, complete data were collected on 588. Loss of subjects was due to changes in class assignments, absenteeism, and mobility within the school system. Analysis of pretest scores of lost subjects indicated no significant differences among the treatment groups, thus indicating apparently comparable attrition among the groups.

A shortened version (every third item, with a total of 30 items) of the Comprehensive Test of Basic Skills (CTBS) was used as a pre- and posttest of achievement. The study was conducted over one school year.

A 2 x 2 factorial design was used, with mastery and teams as the factors. The four treatments were referred to as Focused Instruction (absence of both factors), Teams, Mastery, and Teams and Mastery. Teachers were randomly assigned to treatments, with stratification on school. Most teachers taught only one experimental class, though a few taught more than one.

The curriculum materials were identical in all four treatments: 26 sets of worksheets and quizzes adapted from a general mathematics textbooks. A smiliar schedule of activities (teacher lecture, worksheets, and quiz) was used in all treatments. One complete cycle through these activities usually took one week.

The treatments are described below:

> Focused Instruction: Students worked individually on the worksheets and quizzes. No corrective instruction was provided.

Mastery: Students worked individually, but after the quiz (treated as formative), corrective instruction was provided to those students who did not achieve 80% mastery. A summative quiz was provided after the corrective instruction. Students who mastered the instruction at the formative stage were given enrichment activities pertaining to the same unit.

Teams: Worksheet study was in heterogeneous four-person teams. Quiz scores were compared individually to past averages, and individual improvements were summed as the team scores. Highest scoring teams were recognized in a weekly class newsletter.

Teams and Mastery: Worksheet study was in teams, the quiz was used formatively, corrective instruction was provided in the teams, and a summative quiz was given to all students. The summative quiz was used to form team scores, again based on improvement.

Each teacher was observed to determine whether the major components of the treatment were used in that class. All teachers were observed to be adequately implementing the treatments, "although the quality of implementation varied widely" (p. 730).

4. Findings

ANOVA of the pretest scores of the 588 students in the final sample as well as of the drop-out students indicated no significant difference between treatment groups. The groups thus appeared to be comparable both in terms of initial assignment and in terms of attrition effects.

Nested ANCOVA, with pretest scores as the covariate and class/teacher as the nesting factor, was performed as the main analysis. Significant effects were noted only for Teams ($p < .03$) favoring the use of teams and for the class/teacher effect within Mastery and Teams ($p < .001$). In a separate analysis, no effects were observed for the interaction of trichotomized pretest score (high, middle, low) with treatment.

5. Interpretations

"The results ... do not support the effectiveness of the principal component of group-paced mastery learning techniques" (p. 732). However, there was support for the effectiveness of team work and team reward over and above the regular cycle of presentation/worksheet/quiz. The lack of interaction effect indicated that there was neither advantage nor disadvantage attributable to the addition of the mastery component to the team approach. Further, the large number of students, teachers, and classes involved suggests that the effects are widely generalizable and have considerable ecological validity.

The current results along with other recent research on the mastery learning model suggest that earlier observed benefits of group-based mastery learning may depend on the extra time provided for the mastery learning classes but not for the control classes. This raises the concern that if time needed for corrective instruction does not diminish with continued use of mastery learning, then problems of providing that extra time in real classroom situations need to be confronted.

The lack of effects for the mastery learning treatments might also be attributed to the nature of the sample used. Since the students likely had many serious deficits, the nature of the remediation may have been too narrow and too brief. Finally, the use of group-paced techniques rather than individually prescribed techniques may also have served to diminish the effects of mastery learning.

The positive effects of the teams treatment might have been due to the positive effect of team incentives on student motivation.

Abstractor's Comments

The study is very nicely conducted and reported. Especially convincing is the conservative approach taken both to the data analysis and to the interpretation of the results.

However, given Slavin's copious research on teams, it is surprising that so little is made of the positive effect attributable to the teams factor. The title suggests more or less equal concern for mastery learning and teams, but the authors' interpretations focus on mastery learning almost to the exclusion of teams. One wonders if the management of teams is so cumbersome that the authors do not expect teachers to use teams regularly. The basics of mastery learning are, on the other hand, quite familiar to most teachers, and one might expect the results for the mastery learning factor to be more fully understood. However, if teams are both easy to use and effective, then teachers should be encouraged to use them, and more extensive interpretations of the teams effect seems called for.

The study lasted a year, so it seems reasonable that the novelty of teams would wear off. Yet there is a nagging doubt that the use of teams in mathematics classes and presumably nowhere else in the school may still have constituted an effective treatment just because of its novelty.

There is also a nagging doubt about the appropriateness of the test. The test was a general test, while the instruction in the units was presumably specific for the objectives of the school program. Simply taking every third item in the CTBS doesn't seem adequate for measuring specific objectives.

Of most concern, however, is the notion that the remediation may have been off-target. This is an extremely important idea that seems to be more or less thrown in as an afterthought. Indeed, one cannot

adequately study the effects of the mastery learning approach, which is founded on the notion that corrective action is needed for those students who do not master the material, if one does not know whether the remediation is effective. That means that diagnosis of errors needs to be clear and precise and that remediation needs to be carefully tailored to those errors. Remediation provided in the same way for everyone is almost certainly not going to be effective. Perhaps the lack of effect for mastery learning was due to the ineffectiveness of the remediation, while the team work encouraged peer teaching which was somehow more effective for the subjects.

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Thompson, Alba Gonzalez. THE RELATIONSHIP OF TEACHERS' CONCEPTIONS OF MATHEMATICS AND MATHEMATICS TEACHING TO INSTRUCTIONAL PRACTICE. Educational Studies in Mathematics 15: 105-127; May 1984.

Abstract and comments prepared for I.M.E. by THOMAS GIBNEY, The University of Toledo.

1. Purpose

The primary focus of this study was to investigate teachers' conceptions regarding the subject matter of mathematics and the role of these conceptions in the teaching of mathematics.

2. Rationale

Most research on the effectiveness of mathematics teachers has focused on what a teacher knows and what can be behaviorally demonstrated on the basis of that knowledge. "The question of how teachers integrate their knowledge of mathematics into instructional practice and what role their conception of mathematics might play in teaching have largely been ignored" (p. 105). The author argues that teachers' conceptions (their beliefs, views, and preferences) about mathematics and the teaching of mathematics are important in the teachers' effectiveness to transmit mathematics knowledge from the subject to the student.

3. Research Design and Procedures

Three junior high school mathematics teachers participated in the study. Teacher A had taught junior high school mathematics for ten years and was a mathematics coordinator for a middle school. Teacher B had taught for five years and was in charge of a mathematics component for gifted students at her school. Teacher C had taught junior high school mathematics for three and one-half years and was a mathematics coordinator for her middle school. Teachers A and C were observed

teaching an eighth-grade general mathematics class. Teacher B was observed in a seventh-grade class consisting of "gifted" students.

The method of inquiry used was the case study. Each teacher was observed daily teaching a mathematics class over a period of four weeks. During the first two weeks only observations were conducted. During weeks three and four daily interviews were held following each observed lesson. The mathematics observations and interviews were audio-recorded. The interviews usually lasted for 45 minutes.

Each of the three teachers was also asked to respond in writing to six tasks given at different times throughout the case study. Five tasks sought information regarding each teacher's view about various aspects of mathematics teaching such as goals, objectives, pedagogical practices, student failures and teaching effectiveness. The sixth task was an instrument used to obtain a description of the teachers' view of mathematics in terms of general characteristic qualities of the subject.

The data obtained each day were reviewed in terms of data obtained on previous days. As the case study proceeded, each new analysis provided the focal for subsequent observations and interviews.

4. Findings

The author noted that the case studies showed evidence of differences among the three teachers in the specific beliefs, views, and preferences that they held regarding mathematics and the teaching of mathematics. Teacher A viewed mathematics as a coherent subject consisting of logically interrelated topics. Teacher B regarded mathematics as a challenging subject whose essential processes were discovery and verification. Teacher C indicated a view of mathematics as essentially prescriptive and deterministic in nature. Teachers A and C thought of mathematics as a static body of knowledge and presented content as a finished product.

Teacher A used a more conceptual approach, with mathematics viewed as a set of integrated and interrelated topics, while Teacher C used a more computational approach that saw mathematics as a set of arbitrary rules and procedures for finding answers to specific questions. Teacher B believed that the best way for students to learn mathematics was to engage in its creative processes, so she referred to the heuristic processes of mathematics, discussing them independently of the content being studied.

The teachers' roles in controlling the teaching process differed. Teacher B believed that students learn best by doing; therefore, students' actions would generally control the learning tempo. Teacher A believed it was her responsibility to direct and control all classroom activities. Teacher C viewed her role as a demonstrator of procedures that her students were to use in performing the assignments, and allowed time for the students to work independently on them.

The author reported a sharp contrast among the three teachers with respect to their views about what constituted evidence of mathematical understanding in their students. Teacher A thought it was necessary for students to know the reasons for correct answers and to be able to explain relationships among the topics studied. Teacher B believed that understanding the applications of mathematical topics was sufficient evidence of their ability to integrate their knowledge of facts, concepts, and procedures to a variety of related mathematical tasks. Teacher C believed that mathematical understanding was the ability of her students to follow and verbalize the procedures taught to obtain the correct answers.

The author also found that the teachers' views about planning and preparing for instruction were related to their conception of mathematics. Teacher C saw little benefit in planning her lessons. Teachers A and B regarded the careful preparation of their lessons as an essential first step towards ensuring the quality of instruction. Teacher A's purpose

in planning was to have a logical sequential plan for her explanation of the content, while Teacher B's planning was to help strengthen her knowledge of the topic in order to better handle students' questions. Teacher B used a variety of sources to supplement the textbook. Teacher A followed the textbook closely with no other reference materials. Teacher C's planning was to identify an objective from a published list of objectives and select worksheets to help master the objective.

Another difference reported by the author was the teacher's views about the cognitive goals and objectives of mathematics instruction. Teacher A regarded practical outcomes as more important than disciplinary or cultural outcomes, whereas Teachers B and C saw the disciplinary outcomes as more important than the other two. Teacher B's view was found to be consistent with her instructional mode.

The author reported on two general characteristic qualities of the three teachers' conceptions and behavior, namely integratedness and reflectiveness. Teacher C did not have an integrated conceptual system with regard to mathematics. It was not possible to infer the extent to which Teacher A's beliefs about mathematics were integrated into a coherent system. Teacher B seemed to have a more integrated system of beliefs about mathematics and mathematics teaching then either Teacher A or Teacher C.

Teacher C described her teaching behavior as resembling a tape recorder, admitting her lack of reflectiveness upon her actions and their efforts. Teacher A's reflections were usually based on the ease or difficulty she had in following lesson plans or in eliciting correct student answers. Teacher B reflected on her actions and their effects on her students.

5. Interpretations

The author concluded that this study is a beginning in a research effort to identify key factors that, because of their influence on teachers' instructional practice, may play an important role in their teaching effectiveness. This study, along with previous studies, found that teachers' conceptions are not related in a simple way to their instructional decisions, but rather that the relationships are complex.

The author stated:

> Teachers possess conceptions about teaching that are general and not specific to the teaching of mathematics. They also have conceptions about their students and the social and emotional make-up of their class. These conceptions appear to play a significant role in affecting instructional decisions and behavior. For some teachers, these conceptions are likely to take precedence over other views and beliefs specific to the teaching of mathematics. (p. 125)

Abstractor's Comments

The author is commended for a well-written account of a dissertation and for a contribution in an area of teacher effectiveness that is usually ignored. As indicated by Medley (1982) in Figure 1 there are usually five points in a teacher's career where evaluation might occur.

As stated in the introduction (p. 105), most early research on teacher effectiveness has focused upon the teachers' knowledge of mathematics and recently upon the pupil learning experiences that can be observed. Thus, the training experiences provided in a teacher education program are intended to change the performance competencies of

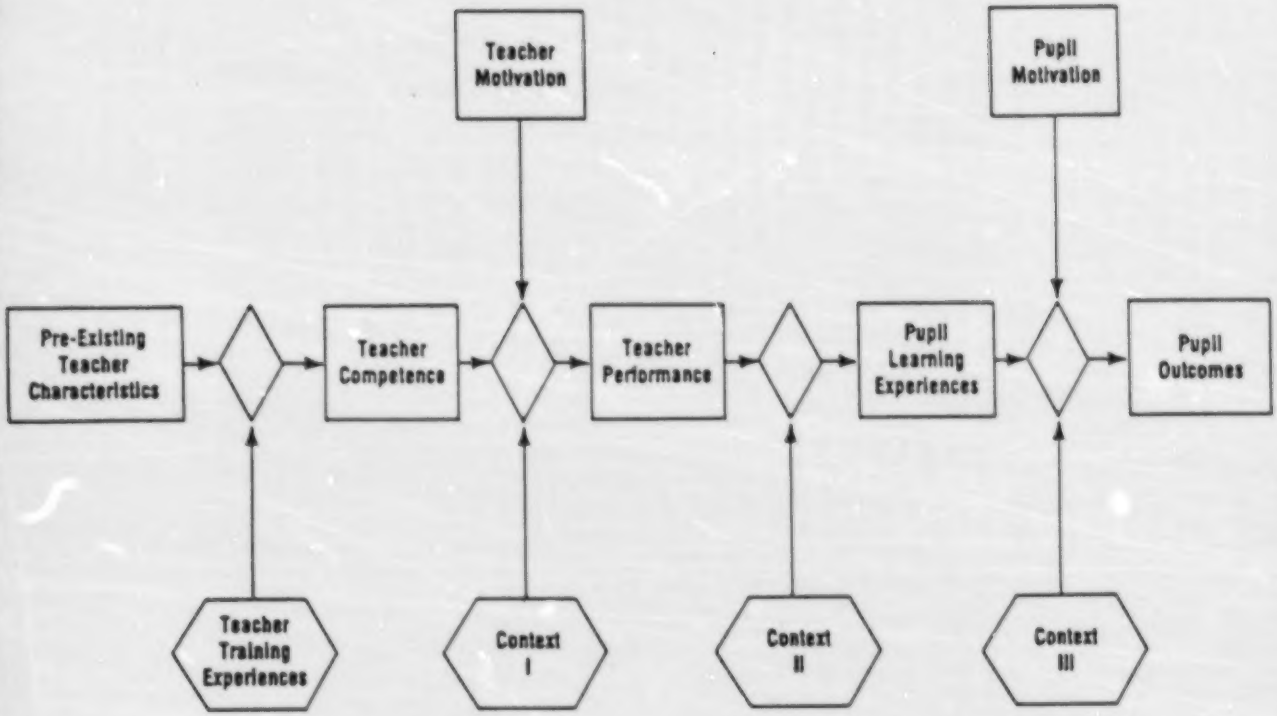

Figure 1. A Conceptual Diagram of the Elements Involved in Teacher Education Research And Evaluation (Medley, 1982)

a teacher in ways that will result in changes in the learning experiences students have, which will in turn change the student outcomes. A perusual of articles relating to teacher effectiveness (Dickson & Wiersma, 1984) indicates that references to mathematics teachers' conceptions (their beliefs, views, and preferences) about mathematics and teaching mathematics are few or ignored. As gatekeepers to the teaching profession all undergraduate teacher preparation institutions should be interested in the findings of this study in terms of entry and exit from their mathematics training programs.

Weaknesses of this study deal primarily with the three subjects used for case studies. With no specific criteria used for subject selection, the small number of three teachers leads the reader to be cautious of the findings. All three subjects appear to have administrative assignments within their schools; therefore, they and their students may not be typical of seventh- and eighth- grade classes. The author mentioned but did not explain the role these three teachers had in a previous pilot study. Was the Hawthorne effect a factor?

The objectiveness of the observer and interviewer in this study is important. The article lacked evidence to convince the reader that this was properly controlled.

Strengths of the article are many. It provided a look at needed research to fill in the Pre-Existing Teacher Characteristics box of Medley's model and to conjecture what role teachers' conceptions play in the Teacher Competence box of the model. The findings suggested differences in what teachers say and how they say it. The suggestions for further research are helpful for others planning to continue research with teachers' conceptions.

The contribution a teacher makes to the effectiveness of a mathematics school program depends upon many factors. This research concerning a teachers' conception of mathematics enhances previous research to provide input into all five points of Medley's model.

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Webb, Noreen M. STABILITY OF SMALL GROUP INTERACTION AND ACHIEVEMENT OVER TIME. Journal of Educational Psychology 76: 211-224; April 1984.

Abstract and comments prepared for I.M.E. by KENNETH E. VOS, College of St. Catherine.

1. Purpose

"This study investigated the stability over time of a) student behavior in small groups and b) the relationships among student and group characteristics, group interaction, and achievement" (p. 211).

2. Rationale

A previous research study (Webb and Cullian, 1983) investigated the stability of interaction in small groups and concluded that student behavior was fairly stable over time. However, this previous research had several limitations including short duration, students changing groups between instructional units, and results obtained from brief observations rather than verbatim reports.

Much of the research on group composition has contrasted uniform-ability groups and mixed-ability groups. Uniform-ability groups seem to be more effective than mixed-ability groups for medium-ability students. Within mixed-ability groups, the medium-ability students tended to be ignored in the interaction by both high- and low-ability students.

3. Research Design and Procedures

a) Sample

Average ability students from three Los Angeles junior high school mathematics classrooms taught by the same teacher were selected for this study. Of the sample of 110 students, 57% were

females and 49% were minority students (black, Mexican-American, Asian-American). Each class contained students from either Grade 7 or 8. Students were assigned to groups by mathematics ability (top 25%, middle 50%, and bottom 25%). They remained in these small groups for the entire experimental period.

b) Procedure

Two personality measures were given at the beginning of the study: Eysenck Personality Inventory and the Intellectual Achievement Responsibility Scale. Also, two teacher-made achievement tests were given: a test on perimeter, area, and volume and a test on measurement of geometric figures using either the nonmetric or metric system.

Small groups (3 or 4 students) worked on two 3-week instructional units with a 3-month interval between units. Unit 1 was on perimeter, area, and volume, while Unit 2 was on the metric system.

Coding of student interactions focused on the major categories of 'gives explanation,' 'receives explanation 'receives no explanation,' 'receives response to procedural questions,' 'gives short-answer feedback,' and 'performs calculations.'

4. Findings

For both Unit 1 and Unit 2, 'giving explanations' was positively related to achievement, while 'receiving no explanation' was negatively related to achievement. For medium-ability students, the high-medium-low group reflected the least number of explanations. For personality measures, the extroversion-introversion subscale showed a significant correlation for Unit 2; extroverted students gave a greater number of

explanations than introverted students. Also those "[S]tudents who perceived that the responsibility for positive and negative achievement outcomes resided in others gave more explanations than students who perceived that the responsibility for positive and negative achievement outcomes resided within themselves" (p. 218). The sex of the student was not related to either achievement or interaction. For Unit 2, analysis of ethnic background showed higher achievement and a tendency to give more explanations by white students.

Student behavior in relation to interaction showed a great deal of instability over time. Fewer explanations were received and the frequency of receiving no explanation after a question dropped during Unit 2 in comparison to Unit 1.

5. Interpretations

a) "Giving explanations seems to be beneficial for achievement, and receiving no explanations seems to be detrimental to achievement" (p. 222).

b) Students who believed that others were responsible for their academic achievement gave more explanations than those who believed they controlled their achievement. Those believing they were in control of their learning may have felt it unnecessary to give explanations since their own efforts determined their achievement level.

c) Previous research showed the advantage of mixed-ability groups over uniform-ability groups to be stable over time. However, this study did not show this stability from Unit 1 to Unit 2. "This result raises the possibility that group composition may be a salient influence on interaction and achievement only for inexperienced groups or for groups who have been assembled for a short time..." (p. 222).

d) Individual student behaviors and interactions were unstable over time. Several plausible explanations for this were given in the discussion section of the report.

e) Previous research seemed to suggest that small student groups functioning in ways beneficial for achievement may not need much monitoring. However, this study may require a modification of this result. The lack of stability of individual student behavior within small groups over time may require more monitoring by the instructor in order "to ensure that students respond to each others' requests for help" (p. 224).

Abstractor's Comments

The researcher, Noreen M. Webb, should be commended for writing a very lucid report of her research. Each major section of the report was complete and focused the reader's attention on the salient points. The tables were designed to incorporate a maximum amount of information with a minimum number of distracting headings or footnotes.

There are at least two minor quibbles with the report:

1) Since the ability test and the achievement tests were integral anchors for either grouping or data analysis, a more complete description should have been given. Also, these tests were teacher-made but not necessarily shown to be reliable measures of ability or achievement.

2) Unit 1 and Unit 2 both involved mathematics as the content area. Of interest to mathematics educators would be a more complete description of these units, especially when Unit 2 reflected significantly different results than Unit 1.

Future or follow-up research of this excellent study could be fertile ground for a doctoral candidate in mathematics education. The report generates at least five "future studies" suggestions in the discussion section. An astute graduate student would do well to reflect on some possible research designs which would attack these unclear trends.

In addition to spin-off studies being generated, either Webb or another mathematics educator should analyze this research and write a brief article for the mathematics classroom teacher. Small group learning is a current bandwagon phenomenon and needs the modulating influence of statements supported by careful research. This report is an excellent foundation for building a bridge between theory and practice in small group instruction.

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